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# Scaling analysis of the magnetic monopole mass and condensate in the pure U(1) lattice gauge theory

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We observe the power law scaling behavior of the monopole mass and condensate in the pure compact U(1) gauge theory with the Villain action. In the Coulomb phase the monopole mass scales with the exponent $\nu_{\rm m} = 0.49(4)$. In the confinement phase the behavior of the monopole condensate is described with remarkable accuracy by the exponent $\beta_{\rm exp} = 0.197(3)$. Possible implications of these phenomena for a construction of a strongly coupled continuum U(1) gauge theory are discussed.

## I. INTRODUCTION

The phase transition between the confinement and Coulomb phases of the strongly coupled pure compact U(1) lattice gauge theory (compact QED) has recently received renewed interest and two of its aspects were investigated in large numerical simulations. First, several attempts have been made to distinguish between the weak first order and second order scenarios for the Wilson action and in the extended coupling parameter space. The question is whether the two-state signal decreasing slowly with increasing lattice volume extrapolates to a nonzero or zero value, respectively, in the thermodynamic limit. (For a recent discussion of this subject and earlier references see Ref. [1]).

Second, a scaling behavior of various bulk quantities and of the gauge-ball spectrum consistent with a second order phase transition and universality has been observed in the vicinity of some points on the manifold separating the confinement and Coulomb phases, outside their narrow neighbourhood in which the two-state signal occurs [2–5]. This suggests that there may exist regions of the parameter space where the transition is continuous, though it may be not such for the particular action used in the simulation.

A continuous transition would allow the construction of a continuum theory. But even if no critical point exists, the theory might be considered as an effective theory, with finite but large cutoff, provided the range of scales at which a second order-like behavior holds is large. The question is then whether such a (possibly effective) continuum U(1) theory would be interesting in some sense, e.g. would it have a phase transition, confinement, etc., in analogy to the lattice regularized theory.

In this paper we address this second aspect and extend the investigation of the scaling behavior to observables related to the magnetic monopoles. To our knowledge this subject has not yet been investigated in a systematic way. But the issue is important, as whatever is interesting in the compact lattice QED is essentially related to the monopoles: The phase transition itself is known to be associated with the occurence of magnetic monopoles being topological excitations of the theory [6–8]. Modifications of the monopole contribution to the action have appreciable consequences for its position [9] and properties [10]. The long distance force in the confinement phase [11,12] and the chiral symmetry breaking [13] are best understood in terms of the monopole condensate. The charge renormalization in the Coulomb phase is due to the antiscreening by monopoles [14–16]. Thus the existence of an interesting effective U(1) theory presumably depends on whether the monopoles persist to play an important role in it, i.e. on the scaling behavior of the monopoles.

Our findings are as follows: In the Coulomb phase we find at various values of the coupling $\beta$ a very clean exponential decay of the monopole correlation function in a large range of distances. This demonstrates the dominance of a single particle state in this correlation function, the monopole, whose mass we determine. The monopole mass extrapolated to the infinite volume, $m_\infty$, scales with the distance from the phase transition as

$$m_\infty(\beta) = a_m(\beta - \beta_c)^{\nu_{\rm m}}, \tag{1.1}$$

where

$$\nu_{\rm m} = 0.49(4). \tag{1.2}$$

This value of the exponent $\nu_{\rm m}$ can be compared with the values for the correlation length exponents $\nu$ obtained for other observables. One of these values is the non-Gaussian value

$$\nu_{\rm ng} \simeq 0.35 \tag{1.3}$$

found for the Lee-Yang zeros [2] at the transition, several gauge balls [3,4] and approximately also for the string tension [3,5]. The other, Gaussian value is

$$\nu_{\rm g} \simeq 0.5. \tag{1.4}$$

It has been observed previously for the scalar gauge ball [3,4] in the confinement phase.

Our main result is that the value (1.2) of $\nu_{\rm m}$ is significantly greater than (1.3) and consistent with (1.4). This implies that monopoles would stay important in any of the conceivable scenarios for a construction of an effective continuum U(1) theory. If such a theory were to be constructed in such a way that masses and other dimensionful observables scaling with the non-Gaussian exponent $\nu_{\rm ng}$ were kept finite nonzero in physical units, then the monopoles in the Coulomb phase would get massless. Even if instead the Gaussian exponent $\nu_{\rm g}$ were used, the monopole mass can be fixed at a finite value in physical units. This second possibility might be particularly suitable in the Coulomb phase, where no other scales are known.

In the confinement phase we have determined the scaling behavior of the monopole condensate extrapolated to the infinite volume, $\rho_\infty$, to be

$$\rho_\infty = a_\rho (\beta_c - \beta)^{\beta_{\rm exp}}, \tag{1.5}$$

where

$$\beta_{\rm exp} = 0.197(3). \tag{1.6}$$

The function (1.5) describes extremely well the data in a broad interval and the scaling behavior of the condensate is thus well established.

However, the value of the magnetic exponent $\beta_{\rm exp}$ alone is not sufficient for considering the continuum limit. For this purpose a renormalised condensate is needed. A natural procedure (like e.g. in the broken $\phi^4$ theory) would be to find a pole in the monopole correlation function in the confinement phase. We find a contribution suggesting such a pole, but the data are consistent with its amplitude, as a function of the lattice volume, extrapolating to zero in the thermodynamic limit. Thus currently the results (1.5) and (1.6) do not allow a conclusion about the condensate in a would-be continuum limit. They constitute only a necessary step in this direction.

The results are presented as follows: In Sec. 2 we summarize known facts about the **Z** gauge theory, which we use in the simulation. It is a dual equivalent to the U(1) lattice gauge theory with the Villain action which we actually investigate. In the Z gauge theory the monopole correlation functions have a form originally found by Fröhlich and Marchetti [17], which is convenient for measurements. Antiperiodic boundary conditions [18,16] are used allowing the consideration of a single monopole in a finite volume in agreement with the Gauss law. These b.c. reduce the magnetic U(1) symmetry [17,16] to $Z_2$. It is pointed out that this symmetry is broken in the confinement phase, which necessitates some caution during simulations.

In Sec. 3 we present our results for the monopole mass in the Coulomb phase and determine its scaling behavior. In Sec. 4 the necessary extrapolation procedure of the monopole mass results to the infinite volume limit is discussed. The finite size effects are sizeable, as monopoles with their Coulomb field are extended objects. The leading term in the volume dependence can be determined from electrostatic considerations, however, and further terms obey a simple ansatz.

In Sec. 5 we present the results for the monopole condensate in the confinement phase and describe the search for a monopole condensate excitation. We discuss our results and conclude in Sec. 6.

## II. THE DUAL FORMULATION OF PURE U(1) LATTICE GAUGE THEORY

### A. Z gauge theory

The partition function of pure U(1) lattice gauge theory

$$Z = \int \prod_{x\mu} \int_{-\pi}^{\pi} d\theta_{x\mu} \, \exp\Big( - \sum_P s(\theta_P) \Big) \tag{2.1}$$

with an action $s(\theta_P)$, $\theta_P$ denoting the plaquette angles, is related to the **Z** (integer) gauge theory by an exact duality transformation [7,17]. In the **Z** gauge theory, the link variables $n_{x\mu}$ have integer values, and the partition function reads

$$Z = \int \mathcal{D}\, n \, \exp\Big( - \sum_P s^*(n_P) \Big), \tag{2.2}$$

$$\int \mathcal{D}\, n = \prod_{x\mu} \sum_{n_{x\mu}=-\infty}^{\infty} , \tag{2.3}$$

with

$$n_P = n_{x\mu} + n_{(x+\hat\mu)\nu} - n_{(x+\hat\nu)\mu} - n_{x\nu} \tag{2.4}$$

being the plaquette integer number associated with a plaquette at position $x$ and orientation $(\mu,\nu)$. In (2.2), $s^*(n_P)$ denotes the dual action. The dual theory is a gauge theory invariant under the transformations

$$n_{x\mu} \mapsto n_{x\mu} + (\nabla_\mu \ell)_x = n_{x\mu} + \ell_{x+\hat\mu} - \ell_x \tag{2.5}$$

with an integer valued function $\ell_x$ of the lattice points $x$. The theories (2.1) and (2.2) are strictly equivalent in the infinite volume limit and they should be comparable in large volumina.

The dual action associated with the Villain action

$$s(\theta_P) = -\log \sum_{k=-\infty}^{\infty} \exp\Big( -\frac{\beta}{2} (\theta_P + 2\pi\, k)^2 \Big) \tag{2.6}$$

is

$$s^*(n_P) = \frac{1}{2\beta} n_P^2 \tag{2.7}$$

whereas the extended Wilson action

$$s(\theta_P) = \beta\,\cos(\theta_P) + \gamma\,\cos(2\theta_P) \tag{2.8}$$

corresponds to the dual action

$$s^*(n_P) = -\log\Big(\int_{-\pi}^{\pi} dz\,\cos(z\,n_P)e^{\beta\,\cos(z)+\gamma\,\cos(2z)}\Big). \tag{2.9}$$

Obviously, for a numerical simulation of the **Z** gauge theory the dual Villain action (2.7) is much more practical than (2.9). This is the reason we choose the Villain action (2.6) in our current work.

### B. Dual correlation functions

The magnetic monopoles in the U(1) lattice gauge theory are described by fields $\Phi(x)$ whose correlation functions are defined by certain modifications of the partition function (2.2) [17]

$$\begin{aligned}&\langle\Phi^\star(y_1)\cdots\Phi^\star(y_j)\Phi(z_1)\cdots\Phi(z_\ell)\rangle\\ &= \frac{1}{Z}\int\mathcal{D}\theta\,\exp\Big(-\sum_P s(\theta_P+X_P)\Big).\end{aligned} \tag{2.10}$$

Here the magnetic (in the U(1) language) flux $X_P$ is generated by the magnetic current density

$$\begin{aligned}\tilde{J}_{x\mu\nu\lambda} = {} & X_{(x+\hat{\lambda})\mu\nu} - X_{x\mu\nu} + X_{(x+\hat{\mu})\nu\lambda}\\ & -X_{x\nu\lambda} + X_{(x+\hat{\nu})\lambda\mu} - X_{x\lambda\mu}\end{aligned} \tag{2.11}$$

with sources $2\pi$ at $z_1,\ldots,z_\ell$ and $-2\pi$ at $y_1,\ldots,y_j$. For given sources, the existence of $X_P$ on finite lattices depends on the topology of the lattice, i.e. on the boundary conditions (b.c.).

In the dual formulation, the expression (2.10) becomes a usual expectation value

$$\begin{aligned}&\langle\Phi^\star(y_1)\cdots\Phi^\star(y_j)\Phi(z_1)\cdots\Phi(z_\ell)\rangle\\ &= \int\mathcal{D}n\,\exp\Big(-i\sum_{x\mu}J_{x\mu}n_{x\mu}\Big)\,\exp\Big(-\sum_P s^\star(n_P)\Big)\end{aligned} \tag{2.12}$$

of a non-local observable [17]

$$\exp\Big(-i\sum_{x\mu}J_{x\mu}n_{x\mu}\Big), \tag{2.13}$$

in which $J_{x\mu}$ is given by the current density (2.11)

$$J_{x\rho} = \frac{1}{6}\tilde{J}_{x\mu\nu\lambda}\epsilon_{\mu\nu\lambda\rho}. \tag{2.14}$$

The observable in (2.12) resembles the non-local expression of a charge operator in terms if its Coulomb field. In the pure U(1) gauge theory an analogous expression would not be gauge invariant. However, in the case of **Z** gauge theory, (2.13) is gauge invariant because the sources of $J_{x\mu}$ are integer multiples of $2\pi$ [17].

### C. The monopole observable

To determine the correlation functions (2.12) in a Monte-Carlo simulation, it is useful to choose the field $X_P$ so that $J_{x\mu}$ vanishes on all links in a certain direction $t$ (referred to as the "time" direction) [16]. Then the observable (2.13) decomposes into factors that are local in $t$ and nonlocal in the "space" directions $\vec{x}$.

Therefore one defines $J_{x\mu}$ for each source located at $(\vec{x}_0, t_0)$ as

$$J_0(\vec{x},t) = 0, \qquad \vec{J}(\vec{x},t) = \vec{B}(\vec{x})\,\delta_{tt_0} \tag{2.15}$$

where $\vec{B}(\vec{x})$ is a solution of

$$\mathrm{div}\,\vec{B}(\vec{x}) = 2\pi\,\delta_{\vec{x}\vec{x}_0} \tag{2.16}$$

in three dimensions. Such a $J_{x\mu}$ in (2.15) yields the monopole field

$$\Phi(\vec{x}_0,t_0) = \exp\Big(-i\sum_{x\mu}J_{x\mu}n_{x\mu}\Big) \tag{2.17}$$

Products of these fields with sources at different positions therefore lead to superpositions of currents $J_{x\mu}$ in (2.12) and (2.13).

In the case of a source (2.15) and (2.16) the boundary conditions in the three space dimensions have to be chosen appropriately. Whereas there is no solution of (2.16) with periodic boundaries, a solution exists for antiperiodic boundaries [16]

$$n_\mu(\vec{x}+L\vec{e}_j,t) = -n_\mu(\vec{x},t) \tag{2.18}$$

on the lattice of spacial extension $L$. That solution can be obtained by the three-dimensional discrete Fourier-transformation of (2.16) on the lattice with respect to $L$-antiperiodic base functions. In terms of the monopole observable (2.17), the boundary conditions (2.18) correspond to magnetic charge conjugation $C$ [16] and are therefore called $C$-periodic.

In principle it would be possible also for periodic b.c. to correlate monopole-antimonopole pairs at large relative distance and determine approximately twice their mass. However, antiperiodic b.c. are much more practical allowing to determine the mass of a single monopole on lattices of moderate sizes.

### D. Dispersion relation and finite size effects

The real and imaginary part of the monopole are even resp. odd with respect to $C$. The observables with definite momentum are $C$-even

$$\Phi_+(\vec{p},t_0) = \sum_{\vec{x}_0}\Re(\Phi(\vec{x}_0,t_0))e^{i\vec{p}\cdot\vec{x}_0} \tag{2.19}$$

and $C$-odd

$$\Phi_-(\vec{p}, t_0) = \sum_{\vec{x}_0} \Im(\Phi(\vec{x}_0, t_0)) e^{i\vec{p}\cdot\vec{x}_0}. \qquad (2.20)$$

Since Fourier-transformation in (2.19) is done with $L$-periodic functions, the momenta are $\vec{p} = (p_1, p_2, p_3)$ with $p_j = 2\pi k_j/L$ and integer $k_j$. In (2.20) Fourier-transformation is applied to $L$-antiperiodic functions, so that the momenta are $p_j = (2k_j + 1)\pi/L$ with integer $k_j$. As a consequence, the whole monopole observable $\Phi = \Phi_+ + \Phi_-$ has no well-defined dispersion relation on finite volume lattices. Only in the infinite volume limit, the momentum spectra degenerate and the monopole mass determined by means the even and odd observables (2.19), (2.20) lead to the same mass. We will have to take this fact into consideration when we examine the volume dependence of our results in Sec. IV.

The monopole masses on finite lattices are determined using

$$m_+ = E_1^+ \qquad (2.21)$$

in the correlation function of the operator (2.19). In the case of (2.20), we assume that the mass is obtained from the energy by the usual particle dispersion relation

$$E_1^2 = m_-^2 + 3\left(2\sin\frac{\pi}{2L}\right)^2. \qquad (2.22)$$

However, for a particle with a Coulomb field in a finite volume this relation is presumably only an approximation. We take the possible corrections into account in a phenomenological extrapolation to the infinite volume. Such a way of extrapolation to the infinite volume is necessary anyhow, as the finite size effects for a particle with extended Coulomb field are large and only partly under analytic control. The results do not change significantly if we use the lattice dispersion relation with $2(\cosh(E)-1)$ instead of $E^2$.

The magnetic U(1) symmetry is reduced to a $\mathbf{Z}_2$ symmetry by imposing $C$-periodic boundary conditions [16,8]. In the confinement phase of pure U(1) gauge theory, this remaining $\mathbf{Z}_2$ symmetry is spontaneously broken. This $\mathbf{Z}_2$ symmetry corresponds to the sign of the $\langle\Phi_+\rangle$ expectation value whereas $\langle\Phi_-\rangle = 0$ due to $C$-antisymmetry. On finite lattices flips between both signs of $\langle\Phi_+\rangle$ can occur, distorting the measurements. Therefore we discard parts of the runs where such flips occur.

## III. RESULTS IN THE COULOMB-PHASE

### A. Simulations and statistics

We simulate the $\mathbf{Z}$ gauge theory with the action (2.7). The boundary conditions are $C$-periodic in spacial directions and periodic in time direction. We use the same heat bath update as the authors of [16] together with a new implementation of the monopole observable (2.17) optimized for vectorization on a Cray T90.

The lattice volumes are $L^3 \cdot T$ with $T = 28$ fixed and different $L$ ranging from 4 to 18. Both monopole operators (2.19) and (2.20) are measured after each 25 update sweeps for each time slice. From these data, the correlation functions are computed. In addition, we determine the action density.

| $\beta$ | $L_{\max}$ | $\tau_{\text{int}}$ |
|---|---|---|
| 0.645 | 18 | 11.0 |
| 0.647 | 18 | 2.5 |
| 0.65 | 18 | 2.3 |
| 0.654 | 18 | 1.9 |
| 0.66 | 16 | 1.3 |
| 0.668 | 16 | 1.1 |
| 0.678 | 16 | 1.0 |

TABLE I. Values of $\beta$ at which the simulations in the Coulomb phase were performed. The lattice sizes are $L = 4, 6, \ldots, L_{\max}$. At each $\beta$, at least $10^4$ measurements have been made. $\tau_{\text{int}}$ is the integrated autocorrelation time of the action density in multiples of 25 sweeps determined on the $L = 8$ lattice.

Table I gives the simulation points in the Coulomb phase, the range $L = 4, 6, \ldots, L_{\max}$ of lattice sizes, and the integrated autocorrelation times of the action on the $L = 8$ lattice. To thermalize the system, we skipped the first $2.5 \cdot 10^3$ (far away from the phase transition) to $2.5 \cdot 10^4$ sweeps (close to the phase transition). At each value of the parameters, at least $10^4$ measurements have been performed.

The restriction of lattice sizes to $L \leq 18$ is mainly due to the costs $\sim L^6$ in the determination of the monopole observable (2.17) because of the two summations over the space volume, one in the exponent, and one in the Fourier-transformation of (2.19,2.20). The statistical errors of the monopole masses are determined by the jackknife method using 16 blocks.

### B. Determination of the monopole mass

We use the observables $\Phi_+$ and $\Phi_-$ with the lowest possible momenta, i.e. $\vec{p} = 0$ for $\Phi_+$ and $\vec{p} = (\pi/L, \pi/L, \pi/L)$ for $\Phi_-$. The values of the monopole mass are obtained from their correlation function which is assumed to have approximately the form

$$\begin{aligned} C_\pm(t) = \langle 0|\, \Phi_\pm^\star(t)\Phi_\pm(0)\,|0\rangle &= |\,\langle 0|\,\Phi_\pm(0)\,|0\rangle\,|^2 \\ &+ |\,\langle 1|\,\Phi_\pm(0)\,|0\rangle\,|^2 \left(e^{-E_1^\pm t} + e^{-E_1^\pm (T-t)}\right). \end{aligned} \qquad (3.1)$$

The first term, the monopole condensate

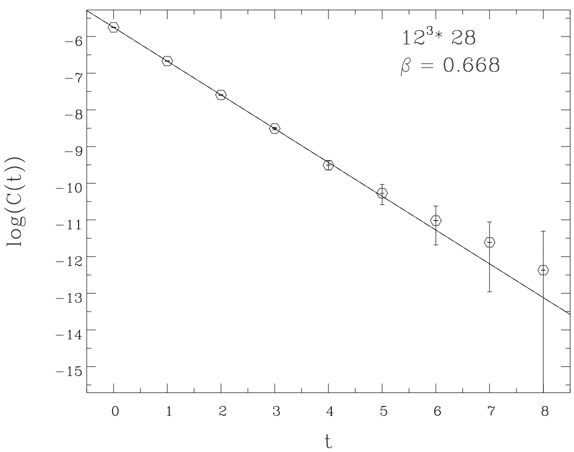


FIG. 1. Logarithmic plot of the correlation function $C_+$ on the $L = 12$ lattice at $\beta = 0.668$. The solid curve is a fit to (3.1) with $m_+ = 0.921(5)$ and $\rho = 0.0003(15)$.

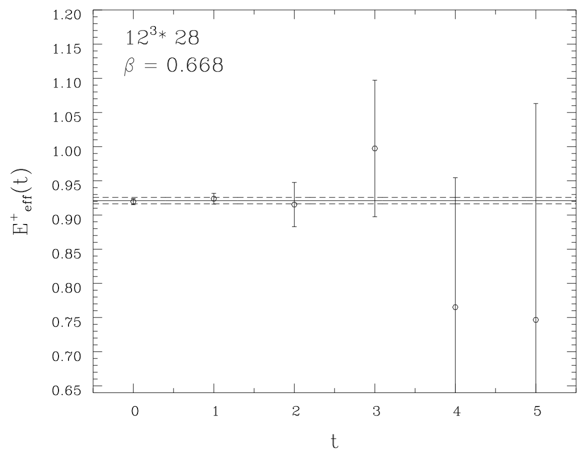


FIG. 2. Effective energies corresponding to the correlation function from figure 1. The horizontal lines show the fit results for $E_1^+$ from (3.1) with $\rho = 0$.

$$\rho = |\, \langle 0 |\, \Phi_+ \,| 0 \rangle \,|, \tag{3.2}$$

is expected to vanish in the Coulomb phase in the infinite volume limit, but should be allowed on finite lattices.

Figure 1 shows as an example the data obtained on the $L = 12$ lattice at $\beta = 0.668$ with the fit by means of the correlation function (3.1). The data in the whole range of $t$ are consistent with a contribution of only one state to the correlation function. To verify this more accurately, we have determined the effective energies $E_{\mathrm{eff}}$ with $\rho$ both free and set equal to zero. The effective energies corresponding to the correlation function from figure 1 are plotted in figure 2. They are stable with respect to $t$. Similar results are obtained for both operators (2.19) and (2.19) at all investigated points in the Coulomb phase for all lattice sizes we have used.

Thus we find that within our numerical accuracy the $t$-dependence of the correlation functions (3.1) can be well described by a one particle contribution. This is remarkable, as the Coulomb phase contains massless photon which could in principle substantially complicate the form of the correlation function (infra particle). It justifies the interpretation of the observed energy as the energy or mass of the magnetic monopole. Furthermore, there is no significant deviation from $\rho = 0$ and therefore from now on we quote only results obtained under the assumption of vanishing condensate.

### C. Scaling behaviour of the monopole mass

From the finite volume results for the masses of the $\Phi_+$ and $\Phi_-$ observables, we estimate the infinite volume mass $m_\infty$ as described below in Sec. IV. The resulting values of $m_\infty$ are given in Tab. II and Fig. 3.

| $\beta$ | $m_\infty$ |
|---|---|
| 0.645 | 0.32(4) |
| 0.647 | 0.46(2) |
| 0.65 | 0.59(3) |
| 0.654 | 0.72(2) |
| 0.66 | 0.88(3) |
| 0.668 | 1.05(3) |
| 0.678 | 1.26(2) |

TABLE II. Extrapolation $m_\infty$ of the monopole mass to the infinite volume with (4.9). The errors are estimated as described in the text.

Figure 3 demonstrates that the monopole mass $m_\infty$ decreases with decreasing $\beta$ and possibly vanishes at the phase transition. The smallest value of $m_\infty$ which we obtained is about $m_\infty \simeq 1/3$ (largest value of the corresponding correlation length is $\xi_{\mathrm{mon}} \simeq 3$). In the range $m_\infty = 0.3 - 1.2$ a scaling behaviour is observed, described by the power law (1.1) with the critical exponent (1.2) and the critical point located at

$$\beta_c^{\mathrm{Coul}} = 0.6424(9). \tag{3.3}$$

The value $\beta_c^{\mathrm{Coul}}$ is consistent with the result for $\beta_c$ obtained for the Villain action in [14,19]. The upper script "Coul" in (3.3) indicates that the position of the critical point has been obtained by means of an extrapolation of some observable from the Coulomb phase. The value of the critical exponent $\nu \simeq 0.5$ has been found earlier to describe the scaling behavior of the mass of a scalar gauge ball in the pure U(1) lattice gauge theory with extended Wilson action [20,4].

It is important to realize why the smallest value of $m_\infty$ we obtained is restricted. As explained in the next sec-

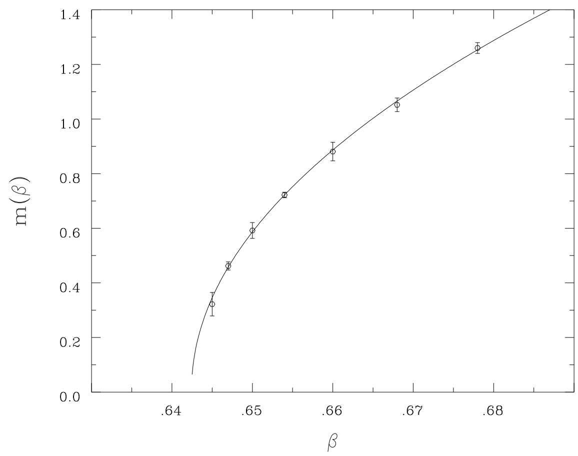


FIG. 3. Scaling of the extrapolated monopole mass $m_\infty$ with the power law (1.1).

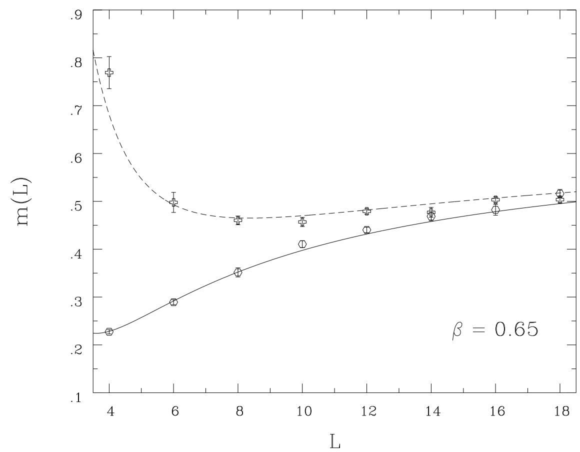


FIG. 4. Dependence of the monopole masses $m_\pm$ on the finite spacial lattice size $L$ at $\beta = 0.65$. The solid and dashed curves for $m_+$ and $m_-$, respectively, correspond to the ansatz (4.8) up to the $1/L^2$ term.

tion, an extrapolation of the finite volume results for the monopole mass to the thermodynamic limit gets gradually more and more difficult as the phase transition is approached. In fact, if the finite volume dependence of the monopole masses were better understood, the phase transition could be further approached without entering the region where metastability occurs. On the $28^4$ lattice metastability occurs only at $\beta \lesssim 0.6439$. An extrapolation of our data using (1.1) to this $\beta$ suggests that this lattice size would allow to reach monopole masses at least as small as 0.2. Thus the range of monopole mass values investigated in this paper is restricted from below by an insufficient understanding of finite size effects and not by the occurence of the two-state signal.

## IV. VOLUME DEPENDENCE OF THE MONOPOLE MASS

Since in the Coulomb phase there exists a long range interaction mediated by the massless photons, a strong dependence of the monopole masses on the finite spacial size $L$ of the lattice is expected. Figure 4 displays the dependence of the masses $m_\pm$ obtained from the correlation functions $C_+(t)$ (solid) and $C_-(t)$ (dashed) on the lattices of size $L$.

The mass $m_+$ which is obtained from the symmetric combination of monopole and antimonopole is smaller than $m_-$. The splitting of masses apparently vanishes if the lattice size $L$ gets large, as expected from the degeneration of the momenta of $\Phi_+$ and $\Phi_-$ in the infinite volume. However, for smaller $L$ the functions $m_\pm(L)$ are rather complicated, $m_-(L)$ being even nonmonotonic, and currently we do not have a sufficient theoretical understanding of this $L$-dependence. To extrapolate to $L = \infty$ we therefore combine the expected asymptotic behaviour of $m_\pm(L)$ with various phenomenological ansätze.

It seems plausible, that at larger $L$, where both masses have already similar values, the long-range Coulomb field of the monopole might dominate the $L$-dependence. So we try to describe $m_\pm(L)$ at large $L$ by the classical energy of a (magnetically) charged particle in a finite volume.

The classical energy of a (magnetic) charge $g$ with a Coulomb potential $\phi(\vec{r}) = g/4\pi r$ in a spherical volume of radius $L$ in continuum is

$$W_{\rm cont}(L) = \frac{\alpha_{\rm mag}}{2}\left(\frac{1}{r_0} - \frac{1}{L}\right), \qquad \alpha_{\rm mag} = \frac{g^2}{4\pi}. \tag{4.1}$$

The diverging classical energy of the point particle has been regularized by a restriction to $r \geq r_0$. The classical consideration gives a characteristic $1/L$ dependence

$$m_\pm(L) = m_\infty - \frac{c_1}{L}. \tag{4.2}$$

with some value of the coefficient $c_1$ depending on the shape of the finite volume, boundary conditions and regularization. There is no difference between $m_+$ and $m_-$ at the classical level. The splitting of masses is probably a quantum mechanical effect, which we have not estimated.

In order to obtain the value of $c_1$ for the cubic lattices used in the simulations, we first carry out a classical computation analogous to (4.1) in the lattice regularization. We use the lattice Coulomb potential

$$\phi_{\rm latt}(\vec{r}) = \frac{g}{L^3}\sum_{\vec{p}\neq 0}\frac{\exp(i\vec{p}\cdot\vec{r})}{2\sum_{\mu=1}^{3}(1-\cos p_j)},$$
$$p_j = \frac{2\pi}{L}n_j, \quad n_j \in \{0,\ldots,L-1\} \tag{4.3}$$

on a $L^3$ cubic lattice with periodic boundary conditions. In the computation of the energy, the gradient is replaced by a finite difference, and the integral over the volume is replaced by a finite sum. We determine the energy numerically. The result is very well approximated by

$$W_{\mathrm{latt}}(L) = 0.2524 - 1.3881\,\frac{\alpha_{\mathrm{mag}}}{2\,L}. \tag{4.4}$$

Compared with (4.1), only the pre-factors have been changed. We do not specify any errors of the coefficients because the statistical errors from the simulations are larger by an order of magnitute. The resulting estimate of the $L$-dependence of the monopole mass on a finite $L^3$ lattice is therefore (4.2) with

$$c_1 = 1.3881\,\frac{\alpha_{\mathrm{mag}}}{2}. \tag{4.5}$$

The drawback of this estimate is the fact that the antiperiodic boundary conditions on the gauge field are not fully respected.

Another way to estimate the value of $c_1$ is to interpret the antiperiodic boundary conditions for the gauge field as a three-dimensional infinite cubic lattice of alternating charges, the distance between nearest neighbours being $L$. For the continuum space this consideration leads to the problem of lattice sums, relevant for various crystaline materials. Taking over the well known results from the condensed matter physics [21] implies that the energy of one monopole in the field of all others is equal to the second term in (4.2) with

$$c_1 = 1.7476\,\frac{\alpha_{\mathrm{mag}}}{2}. \tag{4.6}$$

The numerical factor in (4.6) is the Madelung constant. It determines the classical energy of cubic ion crystals, though its actual calculation requires particular mathematical attention [21]. We prefer (4.6) to (4.5) because it respects the correct boundary conditions for the gauge field.

An estimate of $\alpha_{\mathrm{mag}}$ can be obtained from the numerical analysis of the static potential of U(1) lattice gauge theory with Villain action by using the Dirac relation

$$g \cdot e = 2\pi \qquad \Rightarrow \qquad \alpha_{\mathrm{mag}} = \frac{1}{4\,\alpha_{\mathrm{el}}}. \tag{4.7}$$

We use the values of the renormalized electrical coupling $\alpha_{\mathrm{el}} = e^2/4\pi$ from [14]. Table III contains the values of $\alpha_{\mathrm{el}}$ and $\alpha_{\mathrm{mag}}$ for different $\beta$.

The fact that classical energy considerations suggest a $1/L$ dependence motivates our ansätze in terms of a polynomial in $1/L$. The necessary scale is assumed to be provided by the infinite volume mass $m_\infty$ itself. Our first ansatz is

$$m_\pm(L) = m_\infty + \frac{c_1^{(\pm)}}{L} + \frac{c_2^{(\pm)}}{m_\infty L^2} + \frac{c_3^{(\pm)}}{m_\infty^2 L^3} \cdots \tag{4.8}$$

| $\beta$ | $\alpha_{\mathrm{el}}$ | $\alpha_{\mathrm{mag}}$ | $c_1$ |
|---|---|---|---|
| 0.645 | 0.1836 | 1.3614 | 1.1896 |
| 0.647 | 0.1766 | 1.4159 | 1.2372 |
| 0.650 | 0.1687 | 1.4817 | 1.2947 |
| 0.654 | 0.1606 | 1.5571 | 1.3606 |
| 0.660 | 0.1508 | 1.6580 | 1.4487 |
| 0.668 | 0.1402 | 1.7829 | 1.5579 |
| 0.678 | 0.1293 | 1.9329 | 1.6889 |

TABLE III. Dual renormalized coupling $\alpha_{\mathrm{mag}}$ obtained from (4.7). $c_1$ is the coefficient (4.6) used in the ansatz (4.9).

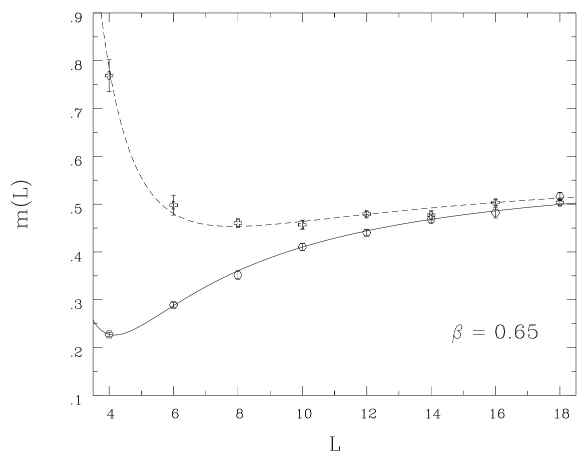


FIG. 5. Dependence of the masses of $m_+$ (solid line) and $m_-$ (dashed line) on the lattice size $L$ at $\beta = 0.65$. The curves correspont to (4.9) up to the term $1/L^3$.

To test the estimate (4.4), different coefficients $c_i^{(+)}$ and $c_i^{(-)}$ are allowed even for $i = 1$. Because of the theoretical uncertainty in this and the other ansätze, we have to be very cautious in estimating the errors of $m_\infty$.

Fig. 4 shows the fits (4.8) upto $1/L^2$. The results obtained for $m_\infty$ become stable if the fit is restricted to data with $L \geq 8$. In this case, $m_\infty$ is not significantly changed if more points at small $L$ are omitted or if $1/L^3$ contributions are included. The coefficients $c_1^{(+)}$ and $c_1^{(-)}$ roughly agree, but differ from the classical value $c_1$ obtained from (4.6) and given in table III by a factor of $1.2 - 1.6$.

Nevertheless, it is possible to fix $c_1^{(+)} = c_1^{(-)} = c_1$ to the classical value and try another ansatz

$$m_\pm(L) = m_\infty - \frac{c_1}{L} + \frac{c_2^{(\pm)}}{m_\infty L^2} + \frac{c_3^{(\pm)}}{m_\infty^2 L^3} \cdots . \tag{4.9}$$

We have used this ansatz with both estimates of the coefficient $c_1$, given in (4.5) and (4.6). It turned out that the results for the monopole mass in the infinite volume are consistent within the error bars. In the following we thus

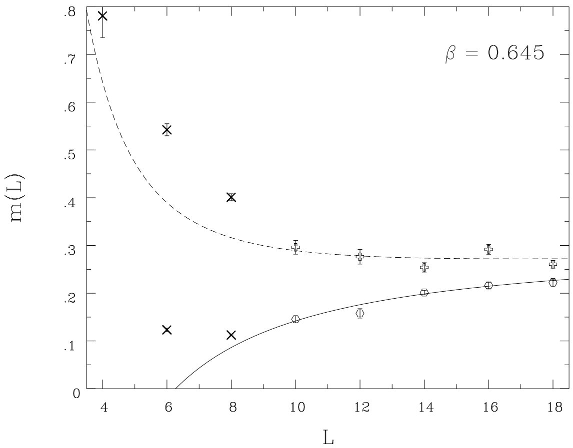


FIG. 6. Dependence of the masses of $m_+$ (solid line) and $m_-$ (dashed line) on the lattice size $L$ at $\beta = 0.645$ which is the point closest to the phase transition. The curves correspond to (4.9) up to $1/L^2$. Data points with $L < 10$ have been excluded from the fits.

describe only the results obtained using the Madelung constant in (4.6).

Fig. 5 shows the same data as fig. 4, but with the functions (4.9) up to $1/L^3$. The fit results $m_\infty$ become stable if the fit includes the $1/L^3$ contributions. A restriction to large $L$ is necessary only for $\beta$ close to the phase transition.

We consider the values of $m_\infty$ obtained in the fit by means of (4.9) with all listed terms as the best determination of the monopole mass in the infinite volume and list the results in table II. To obtain an estimate of the errors of $m_\infty$ obtained in this way, we compare the fit results from (4.9) with and without higher $1/L^k$ terms. Further we omit various number of points at small $L$ from the fits. The error of $m_\infty$ is then estimated from the variation of the fit results under the different conditions. The values of these errors are also given in table II. Using instead of (4.9) the ansatz (4.8) results in the values of $m_\infty$ compatible with those in table II within the listed errors.

Though insufficiently motivated, our extrapolation procedure is rather stable at the $\beta$ values farther from the phase transition. Even the use of the coefficient $c_1$ given in (4.5) instead of (4.6), does not change the results significantly. The extrapolation is most problematic at the point closest to the phase transition, $\beta = 0.645$. Figure 6 shows the $L$-dependence of the $m_\pm$ masses in this case, using (4.6). Since the finite volume mass splitting between $m_+$ and $m_-$ remains significantly nonzero on the largest lattice, the systematic error is probably quite large. From the different fits we estimate it to be about 20% at this $\beta$. In order to reduce this error and to further approach the phase transition, larger lattices and a more reliable extrapolation procedure would be needed.

## V. RESULTS IN THE CONFINEMENT PHASE

### A. Simulations and statistics

Our simulations in the confinement phase are performed with the same algorithm and the same boundary conditions as described in Sec. III A. The lattice volumes are $L^3 \cdot T$ with $T = 24$ fixed and several $L$. At each $\beta$, at least 5000 measurements have been made starting with an ordered system and 5000 starting with a completely disordered system. Before the measurements we have used $2.5 \cdot 10^3$ (away from the phase transition) to $2.5 \cdot 10^4$ (close to the transition) sweeps for thermalization. The monopole operators (2.19) and (2.20) are measured after each 25 update sweeps.

| $\beta$ | $L_{\max}$ | $\tau_{\text{int}}$ |
|---|---|---|
| 0.6 | 12 | 1.1 |
| 0.62 | 14 | 1.2 |
| 0.625 | 14 | 1.3 |
| 0.635 | 14 | 1.3 |
| 0.638 | 12 | 2.6 |
| 0.64 | 16 | 2.9 |
| 0.641 | 16 | 4.2 |
| 0.642 | 16 | 5.9 |
| 0.6425 | 18 | 7.2 |
| 0.643 | 14 | 10 |
| 0.6435 | 18 | 15 |
| 0.6436 | 18 | 30 |
| 0.6437 | 18 | 35 |

TABLE IV. Values of $\beta$ at which the simulations in the confinement phase were performed. The lattice sizes are $L = 4, 6, \ldots, L_{\max}$. $\tau_{\text{int}}$ denotes the integrated autocorrelation time.

Table IV displays the parameters of the simulations and the integrated autocorrelation times $\tau_{\text{int}}$ of the action density in multiples of 25 sweeps. It has been determined on the $L = 8$ lattice at $\beta < 0.6436$ and on $L = 12$ at $0.6436 \leq \beta$. The integrated autocorrelation of the monopole condensate is compatible with these values.

The $\beta$ values are chosen far enough from the phase transition, so that the flips between both phases at the phase transition do not occur in the runs. On the largest lattices we could simulate this leads to the exclusion of runs at $\beta = 0.6438$.

Because of the broken $\mathbf{Z_2}$ symmetry, the finite system sometimes flips between the two possible ground states within the confinement phase. The flip probability increases with decreasing lattice volume and when the phase transition is approached. We observe flips at

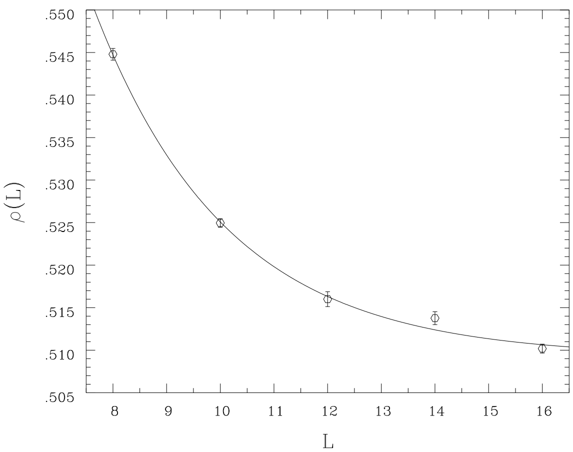


FIG. 7. Dependence of the monopole condensate on $L$ at $\beta = 0.64$. The curve corresponds to (1.5). The scale of the $y$-axis is very fine.

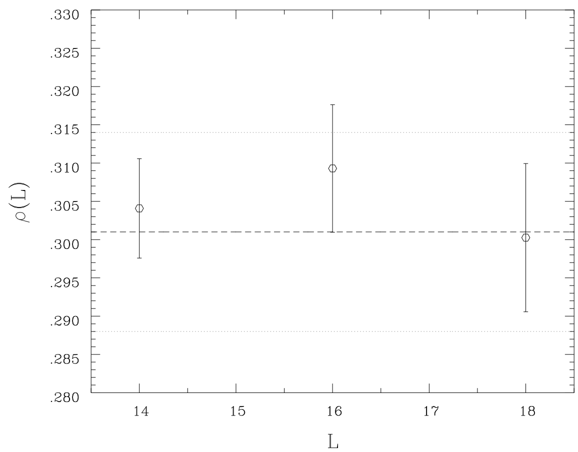


FIG. 8. Dependence of the monopole condensate on $L$ at $\beta = 0.6436$ (close to the phase transition). The horizontal lines indicate the estimated value $\rho_\infty$ and its error.

$\beta \le 0.643$ only on the small lattices with $L \le 8$. At $\beta = 0.6435$, we observe them up to the $L = 14$ and at $\beta = 0.6436, 0.6437$ on all lattice sizes up to $L = 18$. On our largest lattices, the system does not flip more than one or two times during the whole run. In these cases we cut the corresponding parts of the runs to ensure that the unwanted intermediate states do not contribute to the resulting value of the monopole condensate. However, runs with more frequent flips are discarded.

### B. Monopole condensate

The monopole condensate $\langle 0|\,\Phi_+\,|0\rangle$ is measured directly, and its modulus (3.2) can be determined from the correlation function $C_+(t)$ of $\Phi_+$ (3.1). Both methods yield compatible results.

The value of $\rho$ does not depend on the lattice extent $T$ in time direction, but it is weakly dependent on its spacial extent $L$. The $L$-dependence of $\rho$ can be described with an exponential law

$$\rho(L) = \rho_\infty + a\, e^{-bL} \tag{5.1}$$

with constants $a$, $b$ and the infinite volume value $\rho_\infty$. Figure 7 displays $\rho(L)$ at $\beta = 0.64$ with the fit (5.1) used for the extrapolation.

At $\beta = 0.6436$ and $\beta = 0.6437$ the data for the condensate could be obtained only on large lattices due to the ground state flips on smaller ones. As there is no significant $L$-dependence of $\rho$ on the largest lattices, we take the average of the obtained values to represent $\rho_\infty$ instead of using the ansatz (5.1). Figure 8 shows $\rho(L)$ at $\beta = 0.6436$ and the resulting value of $\rho_\infty$.

Table V lists the extrapolated monopole condensate values $\rho_\infty$ at different values of $\beta$.

| $\beta$ | $\rho_\infty$ |
|---|---|
| 0.6 | 0.823(4) |
| 0.62 | 0.730(8) |
| 0.635 | 0.602(5) |
| 0.638 | 0.560(3) |
| 0.64 | 0.509(2) |
| 0.641 | 0.480(6) |
| 0.642 | 0.441(4) |
| 0.6425 | 0.414(5) |
| 0.643 | 0.383(3) |
| 0.6435 | 0.325(10) |
| 0.6436 | 0.300(13) |
| 0.6437 | 0.275(10) |

TABLE V. Extrapolation of the monopole condensate $\rho$ to the infinite spacial volume.

### C. Scaling of the monopole condensate

The $\beta$-dependence of the monopole condensate $\rho_\infty$ is described with an astonishing precision by a simple power law

$$\rho_\infty(\beta) = a_\rho(\beta_c^{\rm conf} - \beta)^{\beta_{\rm exp}} \tag{5.2}$$

with the value (1.6) of the magnetic exponent and

$$\beta_c^{\rm conf} = 0.6438(1). \tag{5.3}$$

Figure 9 shows the scaling of the monopole condensate with the power law (5.2). The logarithmic plot, Fig. 10, is obtained by plotting $\log(\rho(\beta)/a_\rho)$ versus $\log(\beta - \beta_c^{\rm conf})$ with $a$ and $\beta_c^{\rm conf}$ taken as the fit results from (5.2).

We note that the value (5.3) of $\beta_c^{\rm conf}$ and the value (3.3) of $\beta_c^{\rm Coul}$ are consistent within two error bars. The value

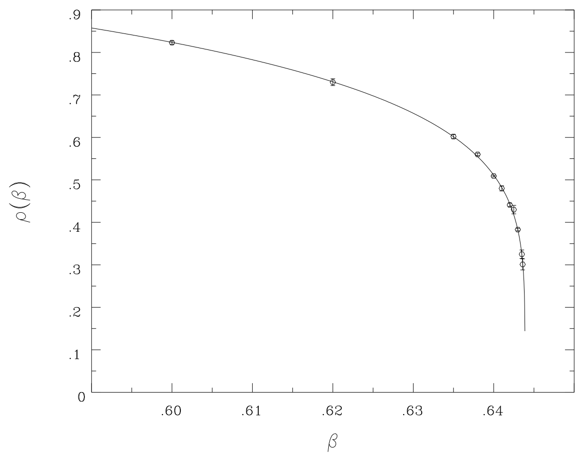

FIG. 9. Scaling behavior of the monopole condensate extrapolated to $L = \infty$ with $\beta$. The curve corresponds to the power law (5.2).

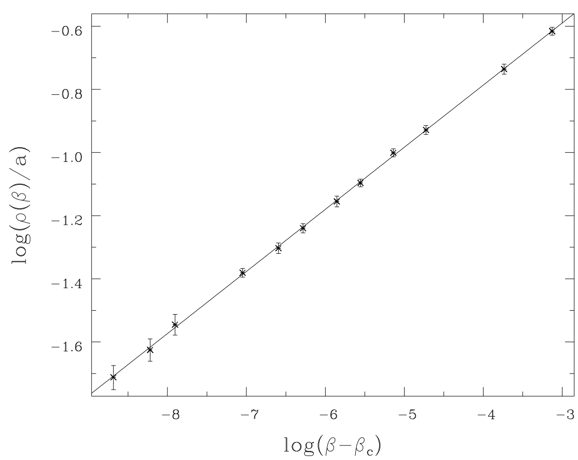

FIG. 10. Scaling of the monopole condensate extrapolated to $L = \infty$ with $\beta$ in a logarithmic diagram. The line corresponds to the power law (5.2).

of $\beta_c^{\text{conf}}$ is much more precise, because the determination of the condensate in the confinement phase is much more precise than that of the monopole mass in the Coulomb phase.

### D. Excited states of the condensate

From the correlation function (3.1) in the confinement phase, one can determine in addition to the monopole condensate the energy and amplitude of a first excited state. If these values gave reasonable results in the infinite volume, they would correspond to a magnetically charged particle-like excitation of the monopole condensate.

There have been attempts to determine the properties of such a state [12]. However, since we use different boundary conditions than the authors of [12], and the remaining symmetry that is dynamically broken is $\mathbf{Z_2}$ in our case instead of the magnetic U(1), our results are difficult to compare with those presented in [12].

We examine the dependence of the energy $E_1^{(\pm)}$ and amplitude squares $a_\pm = |\langle 1|\,\Phi_\pm\,|0\rangle|^2$ on the finite spacial lattice size $L$. The corresponding masses $m_\pm(L)$ obtained from $E_1^{(\pm)}$ by means of the dispersion relations (2.21) and (2.22) show no clear $L$ dependence that would allow an extrapolation to the infinite volume. Also the masses $m_+$ and $m_-$ do not approach each other as $L$ is increased.

| $m_+(L)$ | | | $L$ | | |
|---|---|---|---|---|---|
| $\beta$ | 10 | 12 | 14 | 16 | 18 |
| 0.6 | 1.90(3) | 1.95(2) | | | |
| 0.62 | 1.43(2) | 1.49(2) | 1.53(2) | | |
| 0.635 | 0.93(1) | 0.94(1) | 0.98(2) | | |
| 0.638 | 0.77(1) | 0.78(1) | | | |
| 0.64 | 0.63(2) | 0.635(8) | 0.67(1) | | |
| 0.641 | 0.554(8) | 0.565(7) | 0.582(7) | 0.61(1) | |
| 0.642 | 0.461(7) | 0.479(8) | | 0.50(1) | |
| 0.6425 | 0.396(8) | 0.439(5) | 0.440(7) | 0.44(1) | 0.446(3) |
| 0.643 | 0.31(1) | 0.29(2) | 0.34(1) | | |
| 0.6435 | 0.21(2) | 0.20(3) | 0.18(2) | 0.25(3) | 0.26(2) |
| 0.6436 | | | 0.16(2) | 0.20(2) | 0.19(2) |
| 0.6437 | | | 0.16(2) | 0.14(2) | 0.16(2) |

TABLE VI. Mass $m_+$ of the even excited state in the confinement phase.

| $m_-(L)$ | | | $L$ | | |
|---|---|---|---|---|---|
| $\beta$ | 10 | 12 | 14 | 16 | 18 |
| 0.6 | 2.11(2) | 2.15(2) | | | |
| 0.62 | 1.70(1) | 1.68(2) | 1.69(2) | | |
| 0.635 | 1.23(1) | 12.3(1) | 1.29(1) | | |
| 0.638 | 1.08(1) | 1.09(1) | | | |
| 0.64 | 0.99(1) | 0.98(1) | 0.96(1) | | |
| 0.641 | 0.906(7) | 0.885(6) | 0.890(4) | 0.864(5) | |
| 0.642 | 0.810(7) | 0.801(9) | | 0.781(9) | |
| 0.6425 | 0.764(6) | 0.762(4) | 0.731(8) | 0.716(6) | 0.725(3) |
| 0.643 | 0.719(9) | 0.66(1) | 0.66(1) | | |
| 0.6435 | 0.58(1) | 0.60(2) | 0.51(2) | 0.52(1) | 0.48(3) |
| 0.6436 | | | 0.50(1) | 0.50(2) | 0.48(2) |
| 0.6437 | | | 0.46(2) | 0.45(2) | 0.42(4) |

TABLE VII. Mass $m_-$ of the odd excited state in the confinement phase.

Tables VI and VII show the masses $m_+$ and $m_-$, re-

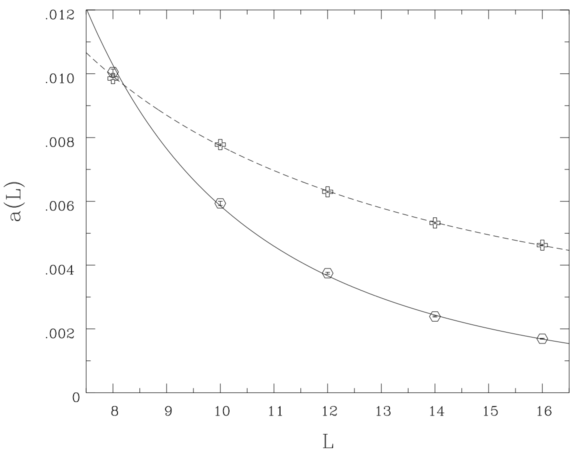

FIG. 11. Amplitudes $a_\pm(L)$ of the $\Phi_+$ (solid) and $\Phi_-$ (dashed) excitations depending on $L$ with the curves (5.4).

spectively, for various $\beta$ and the spacial lattice sizes $L$.

The amplitudes $a_\pm(L)$ decrease as the spacial lattice size is increased. We find that a power law

$$a_\pm(L) = a_\infty^{(\pm)} + \frac{c_\pm}{L_\pm^r} \tag{5.4}$$

gives values of the exponent $r$ in the range $1 \ldots 3$ and values $a_\infty^{(\pm)}$ consistent with zero. Fig. 11 shows as an example the amplitudes $a_\pm(L)$ at $\beta = 0.64$ with fits by the functions (5.4).

These results indicate, that the observed excitations are not present in the infinite volume limit and the corresponding quantities $m_\pm$ should not be interpreted physically as particle masses. For this reason the question of an appropriate renormalization of the monopole condensate remains unanswered.

## VI. CONCLUSION AND DISCUSSION

In the pure U(1) gauge theory with the Villain action we have investigated the scaling behaviour of the monopole mass in the Coulomb phase and of the monopole condensate in the confinement phase. Both observables indicate a critical behavior in the vicinity of the phase transition between these phases, with the values of the corresponding exponents (1.2) and (1.6). Assuming that a continuum theory can be constructed at this phase transition, these results indicate that the monopoles appear also in such a theory. In particular, the monopole mass in the Coulomb phase can vanish or stay finite nonzero in physical units. The Gaussian value (1.2) of the exponent suggests (though not implies) that the corresponding continuum theory may be trivial. This property would then hold also for the scalar QED in the Coulomb phase, as this theory is obtained by duality transformations from the pure compact U(1) theory [7,17].

As for the question of the existence of a continuum limit, in this work we do not contribute to the resolution of the controversy whether the phase transition is of the second or of the weak first order [1]. This was also not our aim in this paper, because for this purpose different methods would be more appropriate. As the values (3.3) and (5.3) of $\beta_c$ obtained from the Coulomb and the confinement phases are consistent within the error bars times 1.5, our results are consistent with the second order. However, a small difference $|\beta_c^{\mathrm{conf}} - \beta_c^{\mathrm{Coul}}| \approx 0.001$ is not excluded by our data, allowing a weak first order transition for the Villain action.

What we want to point out is that, in spite of such a possibility, which actually never can be excluded with full certainty, the pure compact QED on the lattice remains to be a candidate for the construction of an interesting quantum field theory in continuum. The scaling behaviour of the monopole mass and in particular of the monopole condensate is of a high quality, described by a single exponent in the whole accessible region of values of the observables. The same holds also for various other observables [2–5]. This suggests the existence of a continuous phase transition somewhere in the parameter space of possible lattice versions of the pure compact QED. This point of view might be relevant also for the compact lattice QED with fermions [22]. Therefore the compact QED merits further investigation with larger resources and improved methods.

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